Which observable acquires the collective clock-array timescale?

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Tobar, Aspelmeyer and Zych propose to recombine a delocalized mass after its gravitational interaction with a crystal of nuclear clocks. We consider their product-branch model with stationary source positions, initially independent clocks and no direct clock–clock coupling during the hold. Their finite-crystal estimate assigns a collective decoherence time to the mass and to each individual nucleus. The distinction turns on what is recorded. The particle fringe compares two states of the entire crystal. A tagged nucleus, averaged over particle outcomes, samples only its own two branch states. This Comment derives the distinct reduced states and gives a midpoint Ramsey readout whose count parity follows the collective scale. The joint particle-port and clock-count record can test that predicted correlation. [Eqs. (8)–(9) and (18)–(24) of Ref. [1].]

Taking the particle trace

During the hold, take the source positions L, R to be orthogonal and stationary and the clocks initially independent. On branch b, site j evolves to $|u_{bj}\rangle = U_{bj}(t)|+, g\rangle_j$, including its internal state and local motion. The branch Hamiltonian is a sum of site terms, so with a balanced source the state before recombination is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle\bigotimes_j|u_{Lj}\rangle + |R\rangle\bigotimes_j|u_{Rj}\rangle\right).$$

For addressed internal clocks S, define $T_j^{bc} = \mathrm{Tr}_{m_j}(|u_{bj}\rangle\langle u_{cj}|)$ and $g_j = \langle u_{Rj}|u_{Lj}\rangle$. After tracing all local motion and the other clocks, the joint state in the particle path basis is

$$\rho_{PS} = \tfrac{1}{2}\begin{pmatrix} A_L & C_S \\ C_S^\dagger & A_R \end{pmatrix}, \quad A_b = \bigotimes_{j\in S} T_j^{bb}, \qquad C_S = \left(\prod_{j\notin S} g_j\right)\bigotimes_{j\in S} T_j^{LR}. \tag{1}$$

The particle fringe visibility is $|\operatorname{Tr} C_S| = |\prod_j g_j|$. Tracing the particle instead gives $\rho_S = (A_L + A_R)/2$. Every discarded site in a diagonal block contributes $\operatorname{Tr} T_j^{bb} = 1$. Thus adding spectators at fixed local dynamics changes no matrix element of the tagged state, even with unequal distances and branch-dependent local motion. The unconditioned single-clock kernel in Eq. (23) would require a separate source of spectator dependence. [Proposal, Eqs. (14), (18), (23)–(24) and Appendix C.]

The same trace works for a static Gaussian source. For amplitude $f(x)$ in the position basis, the unconditioned internal state is $\int dx\,|f(x)|^2 \bigotimes_{j\in S} T_j^{xx}$. Projection onto a recombined wavepacket retains terms with $x \neq x'$ and the corresponding spectator overlaps. Source motion that mixes positions during the hold, direct clock–clock coupling, or the separately proposed entangled preparation and collective decoding require their own dynamics. [Appendices B and E.]

Product-branch reductions of this form are familiar from spin-environment models. A different interaction graph can give a genuine collective dephasing of one clock. In models of mutually gravitating clocks, the other clocks couple directly to the clock being read. Direct nuclear couplings also arise in the clock model of Beyer et al. [2, 3, 4]

Keeping the particle port

Let $s = \pm 1$ label the measured particle state $(|L\rangle + s e^{i\phi}|R\rangle)/\sqrt{2}$. Both ports occur in the proposed recombination. From Eq. (1) their subnormalized clock states B_s and occurrence probabilities p_s are

$$B_s = \tfrac{1}{4}\left[A_L + A_R + s(e^{i\phi} C_S + e^{-i\phi} C_S^\dagger)\right],$$
$$p_s = \operatorname{Tr} B_s = \tfrac{1}{2}\left[1 + s \operatorname{Re}\left(e^{i\phi} \prod_j g_j\right)\right]. \tag{2}$$

For a clock effect E, $\operatorname{Tr}(E B_s)$ is the joint probability per preparation. Dividing by $p_s > 0$ gives the probability among events at that port. Summing the two joint probabilities gives $\operatorname{Tr}(E\rho_S)$. This weighting matters even when the conditional clock states differ substantially. [Recombination protocol following Eq. (18).]

In the deep-trap redshift limit, choose the local midpoint of the two Ramsey phases as the bright axis. The two branch angles are then $\pm a_j$, where $a_j = t\Delta\omega_j/2$ and $\Delta\omega_j$ is the differential angular frequency. With the mean particle phase compensated, put $c_j = \cos a_j$, $V = \prod_j c_j$ and $K_i = \prod_{j\neq i} c_j$. The unconditioned Ramsey contrast of site i is $|c_i|$. Its probability of a dark outcome, orthogonal to the midpoint bright state, is $d_i = (1 - c_i)/2$. The joint measurement gives

$$p_s = \frac{1 + sV\cos\phi}{2}, \qquad P(s, \mathrm{dark}_i) = \frac{d_i}{2}(1 - sK_i\cos\phi). \tag{3}$$

As a readout extension, at $\phi = 0$, measuring every nucleus at its midpoint recovers the particle port from the parity of the total number k of dark outcomes. For each bright result the two branch amplitudes agree. Each dark result reverses their relative sign. Their sum at the recombiner therefore selects $s = (-1)^k$. Averaged over ports the dark bits are independent with probabilities d_j, because either branch gives the same product distribution. Hence

$$P(k \text{ odd}) = p_- = \tfrac{1}{2}(1 - V), \qquad \mathbb{E}\big[(-1)^k\big] = \prod_j (1 - 2d_j) = V.$$

A classical sampler can generate these bits independently and return their parity as the port. The entire aligned joint table has that description, including rare events.

A broader measurement ambiguity persists for the calibrated pure phase branches. For a local equatorial Ramsey analyzer at angle β_j, outcome $y_j = \pm 1$ has cross-branch matrix element $(c_j + y_j \cos\beta_j)/2$ in the symmetric phase convention. Products of these elements are real. Dephasing the particle in its $\phi = 0$ port basis consequently preserves all joint probabilities for equatorial particle analyzers and product equatorial clock analyzers. The dephased state is separable across the particle–array partition. The equivalence holds for the stated analyzers and phase-only branches. Measurements in additional bases can test coherences outside this set.

Scandium scales for the two reads

Use the proposal's $M = 2.18 \times 10^{-8}\,\mathrm{kg}$, $\omega_c = 1.82 \times 10^{19}\,\mathrm{s}^{-1}$, $D = 450\,\mu\mathrm{m}$ and $\Delta x = 0.1\,\mu\mathrm{m}$. For the nearest on-axis nucleus, the two source distances are D and $D + \Delta x$. Its half-differential phase rate is

$$\dot{a} = \frac{GM\omega_c}{2c^2}\frac{\Delta x}{D(D+\Delta x)} = 7.27345 \times 10^{-17}\,\mathrm{s}^{-1}.$$

At the quoted hold $t = 5\times10^6\,\mathrm{s}$ the ideal tagged contrast loss $1-\cos a$ is 6.61×10^{-20}, corresponding to a midpoint-dark probability of 3.31×10^{-20}. The first half contrast occurs formally at $t = 1.44 \times 10^{16}\,\mathrm{s}$, about 2.88×10^9 times the quoted hold. These numbers isolate the gravitational phase with ideal storage and a calibrated Ramsey reference. [Proposal, parameters below Eq. (8) and redshift in Eq. (17).]

For a finite geometry, take a uniform cube of $N = 10^{23}$ nuclei at density $n = 4 \times 10^{28}\,\mathrm{m}^{-3}$ (side 13.5721 mm). Place the particle on the face normal through its center at the two distances above. Suppressing other motional records, the continuum-density calculation uses

$$\log V = n\int_{\mathrm{cube}} \log\cos a(\mathbf{r})\, d^3r, \qquad a(\mathbf{r}) = \frac{tGM\omega_c}{2c^2}\left(\frac{1}{r_L} - \frac{1}{r_R}\right),$$

where r_b is the distance from the site to source position b. At the same hold, tensor-product Gauss quadrature gives $V = 0.698309$. The port probabilities at $\phi = 0$ are $p_+ = 0.849154$ and $p_- = 0.150846$. The respective conditional dark probabilities for the nearest tagged clock are 5.87×10^{-21} and 1.86×10^{-19}. Their weighted mean is 3.31×10^{-20}. The expected count over the entire cube is $\sum_j d_j \simeq 0.17955$ per preparation. In a separate simple cubic lattice of 50,000,000 × 50,000,000 × 40,000,000 sites at the same density and with the source on the face normal, cell bounds give $0.6963 \le V \le 0.6994$. An odd port in about 15% of ideal preparations is thus compatible with a minute probability at every individual site.

The ^{45}Sc isomer has a population lifetime $T_1 \simeq 0.46\,\mathrm{s}$. Independent unmonitored spontaneous decay alone multiplies the nuclear Ramsey coherence by $e^{-t/(2T_1)}$, giving 0.607 at one lifetime. Liu et al. found no statistically significant coherent forward-scattering signal beyond 2 ms in the tested solids. Their linewidth model gives an ensemble broadening of at least 500 natural linewidths. This measurement leaves the single-site Ramsey T_2 undetermined. Predicting the particle fringe with nuclear decay requires tracing the crystal and all unobserved decay products in the two source branches, including conversion electrons. [5]

In the lossless cube model, shortening the hold to 0.46 s gives an expected dark count of 1.52×10^{-15} per preparation. Even with zero background and perfect detection, a 95% chance of at least one such count requires at least 1.97×10^{15} independent preparations for this readout. With independent decay, the zero-gravity background at one lifetime is about 0.197 dark outcomes per nucleus. Attributing a measured count to the gravitational phase would require a background model and independent calibration. The collective timescale appears in particle interference and, in the aligned measurement, in clock-count parity. The unconditioned contrast of a specified nucleus follows its own differential redshift.

Reproduction and review

The accompanying supplement reproduces the exact Born probabilities, cube integration, finite-lattice enclosure and interrogation estimates with Python. The proof supplement gives the analytic enclosure used in the finite lattice calculation. Code and instructions accompany the Comment at github.com/eprnn/clock-array-ramsey-readout. The calculations concern the idealized model and the stated measurement settings.

Author and AI contributions. Erno Pirinen directed the broader research program and is responsible for this Comment. OpenAI Codex (Astra) drafted and integrated the analysis and manuscript. Two separate Codex reviews independently derived and attempted to falsify the result. A separate AI reviewer (Mesh Neon) proposed the question and reviewed the result and publication scope. Codex prepared the reproducibility files. Anthropic Claude (Opus) independently checked the reported values in its manuscript review. External human specialist review is pending.

References

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