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Can a single quasar proper-motion map measure gravitational-wave handedness?

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Abstract. In the distant-source model with a common temporal filter, a single real proper-motion map has the same Gaussian law for every circular polarization at fixed intensity. The imaginary quadrupole product used in the published conversion is an angular statistic that depends on the chosen pole. Two correctly timed maps give a statistic with a signed helicity response. Unequal observing windows and finite source distances can also carry sign information, which makes calibration specific to the survey.

The real catalogue statistic

Jaraba et al. use a single proper-motion map per catalogue to report signed intervals on the isotropic Stokes parameter V. Their Eq. (3.3) rescales a positive-m quadrupole product by $\frac{0.000876}{(\mu\ \mathrm{as\,yr^{-1}})^2}$ to obtain dimensionless $h_{70}^2\Omega_V$, where $H_0 = 70h_{70}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. We use the documented orthonormal harmonics, with the gradient divided by $\sqrt{\ell(\ell+1)}$ and magnetic mode $\boldsymbol{T} = -\hat{n}\times\boldsymbol{S}$. The first author's thesis explicitly links this basis to the dipole-plus-quadrupole fit. [1] [2] [3]

For either $X = E$ or B, reality of the spatial map imposes $X_{-m} = (-1)^m X_m^*$. The contributions at m and $-m$ to the angular trace $\sum_{m=-2}^{2} E_m B_m^*$ are conjugates, so its imaginary part vanishes in every realization. Equation (3.3) uses

$$S = 2\sum_{m=1}^{2}\mathrm{Im}(E_m B_m^*) = \boldsymbol{e}^{\top} A\boldsymbol{b}, \quad A = 0\oplus\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\oplus\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}, \tag{1}$$

where $\boldsymbol{e} = \left(E_0, \sqrt{2}\,\mathrm{Re}\,E_1, \sqrt{2}\,\mathrm{Im}\,E_1, \sqrt{2}\,\mathrm{Re}\,E_2, \sqrt{2}\,\mathrm{Im}\,E_2\right)$ and $\boldsymbol{b}$ is defined in the same way. For a sky with $E_2 = 1$, $B_2 = i$ in a common motion unit and only their reality partners present, the displayed recipe changes from -2 to $+2$ under a half-turn of the pole. A physical analyser carried with the survey retains its value. Calibrating such an analyser requires its own response to V.

The full one-map distribution at fixed intensity

Take a stationary, isotropic, proper Gaussian tensor background in a single positive-frequency bin. Write the analytic quadrupole responses as $\mathcal{E}$, $\mathcal{B}$ in the real five-mode angular basis. In the distant-source model at fixed intensity the auto-covariances are $p_E\boldsymbol{I}_5$ and $p_B\boldsymbol{I}_5$ and $\langle\mathcal{E}\mathcal{B}^{\dagger}\rangle = i\nu\boldsymbol{I}_5$, with ν in angle squared. Use $e^{-i\omega t}$ and $P_{SS'} = \langle h_S^* h_{S'}\rangle$, so $P_{\times+} = i\frac{V}{2}$. In the pure $V > 0$ case, $h_\times = -ih_+$. For a wave along $+\hat{z}$, the Earth response with Liang et al.'s Eq. (7) conventions, $e_+ = \hat{\varphi}\hat{\varphi} - \hat{\theta}\hat{\theta}$ and $e_\times = \hat{\varphi}\hat{\theta} + \hat{\theta}\hat{\varphi}$, gives $\delta n_\varphi = i\delta n_\theta$. With $\boldsymbol{T} = -\hat{n}\times\boldsymbol{S}$ this gives $\mathcal{B}_m = -i\mathcal{E}_m$, hence $\nu > 0$. Reversing the relative plus/cross sign or the magnetic basis sign reverses the signed read and leaves the null unchanged. [1] [4]

With an identical complex slope response W for every component, the observed real vectors are $\boldsymbol{e} = \mathrm{Re}(W\mathcal{E})$ and $\boldsymbol{b} = \mathrm{Re}(W\mathcal{B})$. Their cross-covariance is

$$\mathrm{Cov}(\boldsymbol{e},\boldsymbol{b}) = \left(\frac{1}{2}\right)\mathrm{Re}\left(|W|^2\ i\nu\right)\boldsymbol{I}_5 = 0. \tag{2}$$

The full ten-dimensional joint law is a centred Gaussian with covariance $\mathrm{diag}\left(s_E^2\boldsymbol{I}_5, s_B^2\boldsymbol{I}_5\right)$ and $s_X^2 = |W|^2\ \frac{p_X}{2}$. It is identical for either pure handedness and for an unpolarized background. For example, the characteristic function of S is $\left(1 + s_E^2 s_B^2 u^2\right)^{-2}$ at every V. The null holds for the full real signal before any harmonic fit. For each GW direction the Earth response matrix R is real, so the input covariance contribution is $|W|^2\ R\,\mathrm{Re}\left(P^{\top}\right)\frac{R^{\top}}{2}$. The V term is purely imaginary in $P^{\top}$ and drops out. Adding stationary, independent bins with a common $W(f)$ across components in each bin preserves this equality at fixed $I(f)$. A sky mask and joint dipole nuisance fit with fixed, helicity-independent noise preserve equality of the input laws. A bound on I can still limit $|V|$ through $|V| \le I$. [4] [5]

A finite measurement that recovers the sign

For a specified frequency $f = \frac{\omega}{2\pi}$ in inverse time, take two secants of the same sky, each of span T, centred at 0 and Δ. The slope transfer at centre t is $W_t = -2i \sin(\pi f T) \frac{e^{-2\pi i f t}}{T}$. For a single tesseral mode,

$$\langle e(0) b(\Delta) \rangle = -\left(\frac{1}{2}\right) |W_0|^2 \, \nu \sin(2\pi f \Delta). \tag{3}$$

With $T = \Delta = \frac{1}{4f}$, three dated position maps suffice for both secants. Define the angularly invariant statistic

$$L = \frac{\boldsymbol{e}(\Delta)^\top \boldsymbol{b}(0) - \boldsymbol{e}(0)^\top \boldsymbol{b}(\Delta)}{5}, \quad \langle L \rangle = 32 f^2 \nu. \tag{4}$$

Both L and its expectation have units of angle squared per time squared. Reversing the order of the centres reverses the mean. Zero helicity and independent sky realizations give zero mean, and the shared position correlates the measurement noise. Çalışkan et al. give an explicit frequency-binned V estimator in Eqs. (2.19), (2.20) and (3.1). The zero-lag and opposite-lag cases distinguish that temporal response from a spatial m selection. [6]

Where a particular catalogue can retain information

Unequal observing windows and the propagation phase at a finite source distance can carry relative phase into fitted slopes. For a concrete synthetic test, a masked survey of 22 sources supplies 44 tangent components to a full-rank, real dipole-plus-quadrupole fit. Use equal-amplitude windows in the distant-source response. With a common centre, the derivative of the whole covariance with respect to V is zero. Stagger the centres on one side of a fixed plane by a quarter-period. At the finest angular integration, the slopes of $\langle S \rangle$ with respect to (I, V) are $(+0.07587, -0.05975)$, compared with $(+0.01806, 0)$ at common times. These are synthetic response units, with the common real-projection and polarization prefactors divided out. Time-reversing the offsets flips the helicity slope and preserves the intensity slope. Carrying the survey and its analyser together under a rotation preserves both. The same score can respond to intensity and helicity.

A finite source distance introduces complex direction-dependent response factors even at a common time. An isotropic finite-distance shell can have a real scalar EB trace. An incomplete survey can mix responses into the oriented statistic S. These mechanisms make a signed response possible in a real catalogue. For a fiducial light-travel distance of 1 Gpc and frequencies of 1–10 nHz, the phase scale $2\pi f \frac{D}{c}$ is about 6.5×10^8 to 6.5×10^9. Finite-distance terms proportional to its inverse are correspondingly small. An actual catalogue response still requires its angular and temporal transfer. Its magnitude and sign would depend on source distances, scan times and directions, weights, the attitude solution and the fit. [7]

Implication for Table 1

For the stationary, isotropic distant-source response with common filtering, the signed conversion in Eq. (3.3) has zero helicity response. Jaraba et al.'s procedure transforms existing catalogue posteriors with an angular product and a scalar frequency conversion. Table 1 therefore lacks a demonstrated calibration to signed V. A catalogue-specific calibration would need the relevant transfer and covariance, with opposite-helicity and intensity-only injections passed through the same fit. That would decide whether the actual observing windows preserve usable sign information. The calculations here use analytic and synthetic inputs. We infer no new sky bound. [1] [2]

Contributions and reproducibility. Erno Pirinen directed the research and is the responsible human author. Astra (Codex) led the analysis and drafted the text. Opus (Claude Code) and Neon (Antigravity) challenged the inference and checked the conventions. AI agents performed the derivations and synthetic checks. External specialist review is pending. Code, proof and replay instructions.

References

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